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Open-Loop Radio Science With a Suppressed-Carrier Signal

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When a suppressed-carrier signal is squared, the carrier reappears in doubled form. An open-loop receiver can be used to deliver a recording of a band-limited waveform containing this carrier, whose amplitude and phase can be tracked by the radio science experimenter.

I. Introduction

In conventional radio science, a spacecraft transmits a sine-wave signal through a medium, which perturbs the signal in phase and amplitude. Inferences about the medium are made from observations of these phase and amplitude variations. Because the present DSN telemetry system uses a residual carrier, open-loop or closed-loop radio science can be carried out in the presence of telemetry modulation. Figure 1 shows an open-loop processor.

The multimegasymbol telemetry system now under development can use a suppressed-carrier signal having no discrete spectral component at the carrier frequency. Doppler phase and signal amplitude can be extracted by a Costas-loop receiver; hence, closed-loop radio science can be carried out as before. In fact, as Ref. 1 shows, this scheme enjoys a 15- to 30-dB advantage over the residual-carrier scheme (with an 80-deg modulation index and no subcarrier) because the latter uses only 1/33 of the total signal power.

Our purpose is to ask whether open-loop radio science can also be carried out with a suppressed-carrier signal. After some tentative investigation into the structure of the optimal receiver for extracting the phase and amplitude of the suppressed carrier, we decided to take a more practical turn. Since the Costas loop has the same performance as a squaring loop (Ref. 2), why not use an open-loop heterodyne receiver to process the square of the residual-carrier waveform? Such a processor is shown in Fig. 2. The recorded waveform contains a doubled version of the carrier.

The comparison between this scheme, which we call squared-suppressed-carrier (SSC), and open-loop reception of a residual carrier (RC), almost duplicates the closed-loop comparison in Ref. 1. The reason for this is that the tape recording of the squared waveform is analyzed by a phase-locked loop in computer software to extract the phase and amplitude of the recovered doubled carrier. In effect, this is nothing but a squaring loop. Nevertheless, we have taken some care to examine exactly what information (and noise) is contained in the square of the suppressed-carrier waveform, as influenced by the dispersion of the medium and of the presquaring filter.

The comparison between RC and SSC is based on the noise variances of the measurements of the phase and amplitude of the perturbed carrier. In the SSC case, there is, of course, no discrete carrier for the medium to perturb, yet we shall see that the reconstructed doubled carrier in the squared waveform contains almost the same information about the medium as the perturbed residual carrier. In fact, if the dispersion of the medium over the bandwidth of the suppressed-carrier

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signal can be neglected, then the SSC receiver of Fig. 2 delivers a signal whose phase is twice the RC phase and whose amplitude is the square of the RC signal amplitude. Therefore, the experimenter must halve the phase and take the square root of the amplitude to obtain phase and amplitude information equivalent to what the RC setup gives. The effect of dispersion is assessed in Section V.

We shall show that the SSC scheme enjoys a considerable signal-to-noise-ratio (SNR) advantage over the RC scheme for the measurement of phase and amplitude. The results can be summarized here. The RC and SSC output tapes (Figs. 1 and 2) are both analyzed by phase-locked loops in software to recover phase and amplitude; let the loop bandwidth in both cases be B_L. Assume that both the RC and SSC received waveforms (the inputs in Figs. 1 and 2) have total signal power A_e^2 and noise spectral density $\frac{1}{2} N_0$. Denoting the estimated carrier phase by $\hat{\psi}_e$, we have

$$\frac{1}{\sigma^2(\hat{\psi}_e)} = \frac{A_e^2}{N_0 B_L} \cos^2\theta \quad \text{(residual carrier)}$$

$$\frac{1}{\sigma^2(\hat{\psi}_e)} = \frac{A_e^2}{N_0 B_L} \beta' \quad \text{(squared suppressed carrier)}$$

For RC, θ is the modulation index. For SSC, β' is a degradation factor, given in Eq. (30) below, caused by the distortion of the presquaring filter $G_i(\omega)$ (Fig. 2) and the noise induced by the squaring operation. These equations are also found in Ref. 1, which compares RC and Costas-loop tracking in the multimegasymbol telemetry environment. For $\theta = 80$ deg and a symbol SNR between 0 dB and 10 dB, we have

$$\cos^2\theta = -15.2 \text{ dB}, -3.1 \text{ dB} < \beta' < -0.8 \text{ dB}$$

for a proper choice of the bandwidth of the presquaring filter (see Fig. 3). It follows that SSC has an advantage of 12 dB to 14 dB over RC for the measurement of phase. The same conclusion holds for the measurement of log-amplitude. This advantage can be used either to decrease the total noise variance of the measurement or to increase the measurement bandwidth B_L.

II. Models

We shall set up conceptual models for open-loop reception of RC and SSC signals. To be most fair to the RC setup, we shall assume that a subcarrier is used. In what follows, ω and λ denote radian frequency.

A. Transmitted Signals

These are

$$\sqrt{2}\cos\theta\cos\omega_c t \quad \text{(residual carrier)} \tag{1}$$

$$\sqrt{2}\, d(t)\cos\omega_c t \quad \text{(suppressed carrier)} \tag{2}$$

where θ = modulation index, $d(t)$ = data modulation, modeled by independent random ±1's with a symbol time T. In our situation, 0.1 MHz $\leqslant 1/T \leqslant$ 30 MHz. The spectral density of $d(t)$ is

$$S_d(\lambda) = T\,\mathrm{sinc}^2(\tfrac{1}{2}\lambda T) \tag{3}$$

The random Fourier expansion of the stationary process $d(t)$ can be written symbolically* as

$$d(t) = \int_{-\infty}^{\infty} D(\lambda) e^{i\lambda t}\, d\lambda \tag{4}$$

B. External Medium and Received Waveforms

This is what the radio science experimenters want to probe. We represent it by a time-varying transfer function $G_e(\omega, t)$, the "external filter." It operates as follows: Any RF exponential $Ae^{i\omega t}$ emerges from the medium as the narrowband waveform $G_e(\omega, t)Ae^{i\omega t}$, and the principle of superposition holds for linear combinations of complex exponentials. Further, $G_e(\omega, t)$ is scaled so that it accounts for the power transmitted by the spacecraft, the space loss, and the gains of the transmitting and receiving antennas. Thus, the received residual-carrier waveform is

$$\frac{1}{\sqrt{2}}\cos\theta\, G_e(\omega_c, t)\, e^{i\omega_c t} + \text{c.c.} + n_w(t)$$

$$= \sqrt{2}\cos\theta\, A_e(t)\cos(\omega_c t + \psi_e(t)) + n_w(t) \tag{5}$$

where c.c. means complex conjugate, $G_e(\omega_c, t)$ has amplitude $A_e(t) > 0$ and phase $\psi_e(t)$, and $n_w(t)$ is white Gaussian noise with spectral density $\frac{1}{2} N_0$. We presume that the desired radio science information is contained in $A_e(t)$ and $\psi_e(t)$.

*Actually, $D(\lambda)\, d\lambda$ represents a process with orthogonal increments. In the Appendix, we shall approximate $d(t)$ by a process with a discrete Fourier expansion.

Applying G_e to Eq. (2), with $d(t)$ given by Eq. (4), we obtain the received suppressed-carrier waveform as

$$\frac{1}{\sqrt{2}} d_e(t) e^{i\omega_c t} + \text{c.c.} \tag{6}$$

where

$$d_e(t) = \int_{-\infty}^{\infty} G_e(\omega_c + \lambda, t) D(\lambda) e^{i\lambda t} d\lambda \tag{7}$$

is the data waveform after distortion by the baseband filter $G_e(\omega_c + \lambda, t)$. If

$$G_e(\omega_c - \lambda, t) = \overline{G_e}(\omega_c + \lambda, t)$$

were to hold (the bar means complex conjugate), then $d_e(t)$ would be real. In general, $d_e(t)$ is complex.

C. Open-Loop Receivers

Stripped to its conceptual essentials, an open-loop receiver for the residual-carrier waveform Eq. (5) is shown in Fig. 1. The heterodyne frequency f_h is chosen such that $0 < f_c - f_h < B_v$, where B_v is the "video" bandwidth, ranging from 100 Hz to 50 kHz in the DSN Multi-Mission Receiver (Ref. 3). The video waveform is sampled at $2B_v$ and recorded.

To process the suppressed-carrier waveform Eq. (6), the receiver of Fig. 2 is suggested. Before being squared, the waveform passes through an "internal" bandpass filter G_i with a nominal passband $f_c \pm B$. We shall show that the squared waveform contains a discrete component at $2f_c$ containing the radio science information. Mixing this with $2f_h$ gives another video waveform with bandwidth $2B_v$. This processor will be called the *squared-suppressed-carrier receiver*.

III. Analysis of the Squared-Suppressed-Carrier Receiver

Let $G(\omega, t) = G_e(\omega, t) G_i(\omega)$, the overall filter. In analogy with Eq. (7), set

$$d_G(t) = \int_{-\infty}^{\infty} G(\omega_c + \lambda, t) D(\lambda) e^{i\lambda t} d\lambda \tag{8}$$

The output of the internal filter in Fig. 2 is $x(t) = s(t) + n(t)$, where

$$s(t) = \frac{1}{\sqrt{2}} d_G(t) e^{i\omega_c t} + \text{c.c.} \tag{9}$$

$$n(t) = g_i(t) * n_w(t) \tag{10}$$

and $g_i(t)$ is the impulse response of G_i. Then

$$x^2(t) = s^2(t) + 2s(t)\, n(t) + n^2(t) \tag{11}$$

Let us first examine the S × S signal $s^2(t)$. In the Appendix it is shown that $s^2(t)$ is the sum of two components, one with discrete spectral lines and the other with a spectral density. The discrete component has spectral lines at frequencies $2f_c + n/T$ ($n = 0, \pm 1, \pm 2, \cdots$) plus a DC term that we ignore.

The central component at $2f_c$, the reconstructed doubled carrier, is the narrowband waveform

$$1/2\, Y(0, t) e^{i2\omega_c t} + \text{c.c.} \tag{12}$$

where

$$Y(0, t) = \int_{-\infty}^{\infty} G(\omega_c + \lambda, t) G(\omega_c - \lambda, t) \operatorname{sinc}^2 (\tfrac{1}{2} \lambda T) \frac{T d\lambda}{2\pi} \tag{13}$$

(a special case of the function $Y(\omega, t)$ defined in the Appendix). If $G(\omega, t)$ were equal to $G_e(\omega_c, t)$ for all ω, then, because $\operatorname{sinc}^2 (\tfrac{1}{2}\lambda T)$ integrates to 1, Eq. (13) would become $Y(0, t) = G_e^2(\omega_c, t)$. This means that Eq. (12) would carry the same radio science information as the residual-carrier signal in Eq. (5) except that here the carrier would appear in doubled form; Eq. (12) would be

$$A_e^2(t) \cos (2\omega_c t + 2\psi_e(t)) \tag{14}$$

In practice, G will have dispersion due to the external medium and the receiver, especially the $\pm B$ bandpass. The actual Eq. (13) gives $Y(0, t)$ as a weighted average of $G(\omega_c + \lambda, t) G(\omega_c - \lambda, t)$. We shall say more about this in Section V.

The spurious S × S components at $2f_c + n/T$ ($n \neq 0$) will be ignored because we assume that the video bandwidth $2B_v$ is less than $1/T$. As with Costas loop tracking, one must be careful not to mistake a spur for the main component (Ref. 4).

The continuous-spectrum component of $s^2(t)$, the S X S noise, is almost small enough to be ignored in this study. Nevertheless, we shall set down its spectral density here for the record. Let $\omega_1 = 2\pi/T$. Define the function

$$\Phi(0, \lambda, t) = \sum_{k=-\infty}^{\infty} G(\omega_c + \lambda + k\omega_1, t) G(\omega_c - \lambda - k\omega_1, t)$$

$$\times \operatorname{sinc}^2 (\tfrac{1}{2}(\lambda + k\omega_1)T) \tag{15}$$

(a special case of the function $\Phi(\omega, \lambda, t)$ defined in the Appendix). This function (of λ) is ω_1-periodic, and its average over a period is just $Y(0, t)$. The spectral density of the S X S noise at $2f_c$ is now

$$S_{ss}(2\omega_c, t) = \frac{T}{2} \int_0^{\omega_1} |\Phi(0, \lambda, t) - Y(0, t)|^2 \frac{T d\lambda}{2\pi} \tag{16}$$

which is $T/2$ times the mean-square variation of Φ over a period. This is a time-dependent, short-term spectral density, valid for integration times that are short compared to the fluctuations of $G_e(\omega, t)$.

Equation (16) is equivalent to Eq. (17) of Ref. 5.

The spectral densities of $s(t)$ and $n(t)$ in Eqs. (9) and (10) for frequencies near f_c are

$$S_s(\omega_c + \lambda, t) = \tfrac{1}{2}|G(\omega_c + \lambda, t)|^2 \, T \operatorname{sinc}^2 (\tfrac{1}{2} \lambda T)$$

$$S_n(\omega_c + \lambda) = \tfrac{1}{2} N_0 |G_i(\omega_c + \lambda)|^2 \tag{17}$$

Therefore, the spectral densities at $2f_c$ of the S X N noise $2s(t)n(t)$ and the N X N noise $n^2(t)$ are (Ref. 6)

$$S_{sn}(2\omega_c, t) = N_0 \int_{-\infty}^{\infty} |G(\omega_c + \lambda, t) \, G_i(\omega_c - \lambda)|^2$$

$$\times \operatorname{sinc}^2 (\tfrac{1}{2} \lambda T) \frac{T d\lambda}{2\pi} \tag{18}$$

$$S_{nn}(2\omega_c, t) = \tfrac{1}{2} N_0^2 \int_{-\infty}^{\infty} |G_i(\omega_c + \lambda)$$

$$\times G_i(\omega_c - \lambda)|^2 \frac{d\lambda}{2\pi} \tag{19}$$

(Because $n(t)$ is Gaussian, the spectral density of $n^2(t)$ is $2S_n(\omega)*S_n(\omega)$.)

IV. Comparison of Open-Loop Receivers

A. Ratio of Power to Noise Spectral Density

Before proceeding to the comparison of the noise variances of phase and log-amplitude, it is convenient to compute power/noise ratios for the RC and SSC receivers. For the RC receiver, the ratio of signal power to one-sided noise spectral density (NSD) of the waveform Eq. (5) is

$$\frac{\text{Power}}{\text{NSD}} = \frac{A_e^2}{N_0} \cos^2 \theta \quad \text{(residual carrier)} \tag{20}$$

We have dropped the dependence of A_e on time.

To get a simple expression for the squared-suppressed-carrier (SSC) receiver, let us assume that

$$G_e(\omega_c \pm \lambda, t) = G_e(\omega_c, t), \quad (|\lambda| < 2\pi B) \tag{21}$$

$$G_i(\omega_c + \lambda) = 1, \quad (|\lambda| < 2\pi B)$$

$$= 0, \quad \text{otherwise} \tag{22}$$

This means that we are neglecting the dispersion of the external filter over the passband of the internal filter, which is assumed ideal. Then the doubled reconstructed carrier, Eq. (12), is

$$A_e^2(t) \, \alpha \cos(2\omega_c t + 2\psi_e(t)) \tag{23}$$

where

$$\alpha = \int_{-2\pi BT}^{2\pi BT} \operatorname{sinc}^{2\,i}(\tfrac{1}{2} x) \frac{dx}{2\pi} \tag{24}$$

This α (the same as in Ref. 1) measures the signal loss due to the distortion of $d(t)$ by the receiver.

The noise spectral densities from Eqs. (18), (19), and (16) become (with time dependence dropped)

$$N_0 A_e^2 \alpha \qquad (\mathrm{S} \times \mathrm{N})$$

$$N_0^2 B \qquad (\mathrm{N} \times \mathrm{N}) \qquad (25)$$

$$\tfrac{1}{2} A_e^4 T\mu_2 \qquad (\mathrm{S} \times \mathrm{S})$$

where μ_2, a function of BT, is the mean-square oscillation of Φ/A_e^4 over a period $2\pi/T$. In the cases given below, the S X S density is almost negligible.

Now let $R = A_e^2 T/N_0$, the symbol SNR. Equations (23) and (25) give

$$\frac{\text{Power}}{\text{NSD}} = \frac{\tfrac{1}{2} A_e^4 a^2}{2(N_0 A_e^2 \alpha + N_0^2 B + \tfrac{1}{2} A_e^4 T\mu_2)}$$

$$= \frac{A_e^2}{N_0} \tfrac{1}{4} \gamma \quad \text{(squared-suppressed-carrier)} \qquad (26)$$

where the degradation factor γ is given by

$$\gamma = \frac{R\alpha^2}{R\alpha + BT + \tfrac{1}{2} R^2 \mu_2} \qquad (27)$$

which is the same as β' of Ref. 1 (see Eq. (30) below) except for the small S X S noise term $\tfrac{1}{2} R^2 \mu_2$. The next subsection will show why the factor 1/4 in Eq. (26) is not counted as a degradation.

B. Amplitude and Phase Noise Variances

Consider now the actual quantities of interest, namely A_e and ψ_e. Let us suppose that for both RC and SSC the phase of the signal in the video waveform is tracked (in nonreal time) by a phase-locked loop with one-sided bandwidth B_L. For RC (the waveform of Eq. (5) heterodyned to video), the variance of the loop phase estimate $\hat{\psi}_e$ is given by

$$\frac{1}{\sigma^2(\hat{\psi}_e)} = \frac{A_e^2}{N_0 B_L} \cos^2\theta \quad \text{(residual carrier)} \qquad (28)$$

For SSC we must remember that the signal being tracked is the doubled carrier Eq. (23) (heterodyned to video). Thus, the usual linear-loop phase error variance is really the variance of $2\hat{\psi}_e$. Computing the variance of $\hat{\psi}_e$ introduces a factor of four that cancels the 1/4 in Eq. (26) and gives

$$\frac{1}{\sigma^2(\hat{\psi}_e)} = \frac{A_e^2}{N_0 B_L} \beta' \quad \text{(squared-suppressed-carrier)} \qquad (29)$$

where, as in Ref. 1 (also see Refs. 2 and 5),

$$\beta' = \frac{R\alpha^2}{R\alpha + BT} \qquad (30)$$

The S X S noise, small to begin with, has been suppressed by the coherent phase detection.

Because the SSC receiver does not have to deliver telemetry, the presquaring filter bandwidth $2B$ can be chosen to minimize the phase noise variance. It can be assumed that the symbol rate $1/T$ is set so that the nominal symbol SNR $R = A_e^2 T/N_0$ has some reasonable value, say between 0 dB and 10 dB. Given R, one can choose BT to maximize β'. (Recall that α is a function of BT from Eq. (24).) The resulting BT and β' are given as functions of R in Figs. 3 and 4 for $-4\ \mathrm{dB} \leqslant R \leqslant 12\ \mathrm{dB}$. (At a value of R slightly more than 12 dB, the optimal BT jumps discontinuously to a value greater than 1.) Also shown is the residual carrier suppression $\cos^2\theta$ for several values of the modulation index θ; the ratio $\beta'/\cos^2\theta$ is the advantage of SSC over RC.

Let the amplitude of the signal in the video waveform be extracted by a coherent amplitude detector (Fig. 5) whose low-pass filter has one-sided noise bandwidth B_A. With perfect phase tracking, the noise variance of the amplitude estimate $\hat{A}$ is

$$\sigma^2(\hat{A}) = N_A B_A \qquad (31)$$

where N_A is the one-sided spectral density of the baseband noise from the amplitude detector.

For RC we have $A = A_e \cos\theta$, $N_A = N_0$; hence

$$\frac{A_e^2}{\sigma^2(\hat{A}_e)} = \frac{A_e^2}{N_0 B_A} \cos^2\theta \quad \text{(residual carrier)} \qquad (32)$$

For SSC we have $A = (\alpha/\sqrt{2})\, A_e^2$, $N_A = 2N_0 A_e^2 \alpha + 2N_0^2 B + 2A_e^4 T\mu_2$. Since $\sigma(\hat{A})/A \approx 2\sigma(\hat{A}_e)/A_e$, the ¼ factor in Eq. (26) again goes away, and

$$\frac{A_e^2}{\sigma^2(\hat{A}_e)} \approx \frac{A_e^2}{N_0 B_A} \frac{R\alpha^2}{R\alpha + BT + R^2\mu_2}$$

(squared suppressed carrier) (33)

The degradation factor in Eq. (33) is the same as γ (Eq. (27)) except that the small S X S noise term has been doubled by the coherent detection. It can still be ignored in this context. Again, one can use Fig. 4 to compare SSC to RC.

V. The Effect of Dispersion

The suppressed-carrier signal, although its bandwidth may be as much as 60 MHz, is still a narrowband signal because the carrier frequency is at least 2.3 GHz. Dispersive effects of the medium may therefore be negligible. Nevertheless, let us measure the dispersion of the external filter $G_e(\omega, t)$ near ω_c by the function

$$H(\lambda, t) = \frac{G_e(\omega_c + \lambda, t)}{G_e(\omega_c, t)} \tag{34}$$

Then $G(\omega_c + \lambda, t) = G_e(\omega_c, t)\, G_i(\omega_c + \lambda)\, H(\lambda, t)$. If $H(\lambda, t)$ does not depend on t, then by Eq. (13) the complex amplitude $Y(0, t)$ of the reconstructed doubled carrier is still proportional to $G_e^2(\omega_c, t)$. In any case, assume that for $\omega_c + \lambda$ in the passband of G_i we have the development

$$H(\lambda, t) = 1 + h_1(t)\lambda + h_2(t)\lambda^2 + \cdots \tag{35}$$

Assume that $G_i(\omega, t)$ is known. Neglecting the remainder in Eq. (35), we have

$$Y(0, t) = G_e^2(\omega_c, t) \left[\int_{-\infty}^{\infty} G_i(\omega_c + \lambda) \times G_i(\omega_c - \lambda) \operatorname{sinc}^2(\tfrac{1}{2}\lambda T) \frac{T d\lambda}{2\pi} + \frac{4(2h_2 - h_1^2)}{T^2} \int_{-\infty}^{\infty} G_i(\omega_c + \lambda) \times G_i(\omega_c - \lambda) \sin^2(\tfrac{1}{2}\lambda T) \frac{T d\lambda}{2\pi} \right] \tag{36}$$

The second term carries the dispersive effect. Although one cannot measure the dispersion by observing $Y(0, t)$, one could use prior bounds on h_1 and h_2, and their variation with time, to estimate how much the ratio $Y(0, t)/G_e^2(\omega_c, t)$ varies with time.

VI. Conclusions

By squaring a suppressed-carrier signal, it is possible to provide the radio-science experimenter with a tape containing the reconstructed doubled carrier. For measuring amplitude and phase, this scheme has typically a 12-dB to 14-dB advantage over the present residual-carrier scheme.

If the dispersion of the medium over the presquaring filter passband is neglected, then the reconstructed doubled carrier yields the same information as the residual carrier; one merely takes the square root of the amplitude and halves the phase. There is a phase ambiguity of π instead of 2π. This is of no consequence if the phase is tracked continuously.

It is anticipated that the effect of media dispersion on the doubled carrier reconstruction will be small because the bandwidth of the suppressed-carrier signal is small compared to, for example, the difference between S band and X band. The effect of this dispersion can be estimated.

References

1. Lesh, J. R., "Tracking loop and modulation format considerations for high rate telemetry," *DSN Progress Report 42-44*, Jet Propulsion Laboratory, Pasadena, California, 1978.

2. Lindsey, W. C., and Simon, M. K., *Telecommunication Systems Engineering*, Prentice Hall, 1973.

3. Donnelly, H., and Nishimura, H., "Multi-Mission Receiver (MMR)," *DSN Progress Report 42-52*, Jet Propulsion Laboratory, Pasadena, California, 1979.

4. Hedin, G. L., Holmes, J. K., Lindsey, W. C., and Woo, K. T., "Theory of False Lock in Costas Loops," *IEEE Trans. Commun.*, COM-26, 1-12, Jan. 1978.

5. Simon, M. K. and Lindsey, W. C., "Optimum Performance of Suppressed Carrier Receivers with Costas Loop Tracking," *IEEE Trans. Commun.*, COM-25, 215-226, Feb. 1977.

6. Davenport, W. B., Jr. and Root, W. L., *An Introduction to the Theory of Random Signals and Noise*, McGraw Hill, 1958.

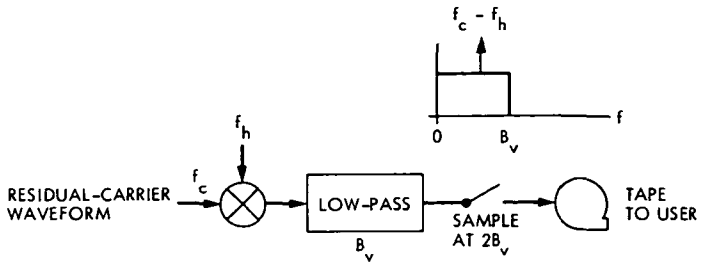

Fig. 1. Conceptual model of residual-carrier, open-loop receiver

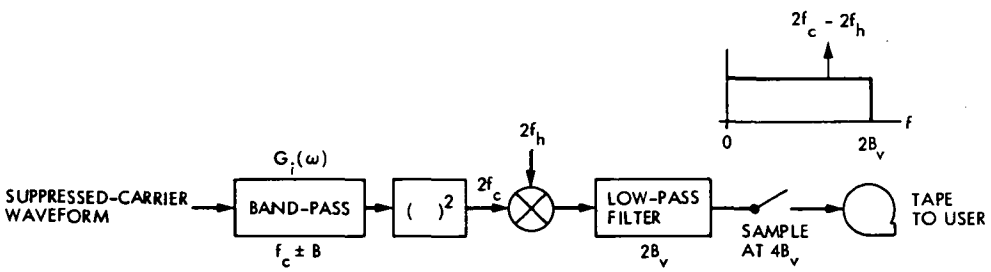

Fig. 2. Conceptual model of square-suppressed-carrier, open-loop receiver

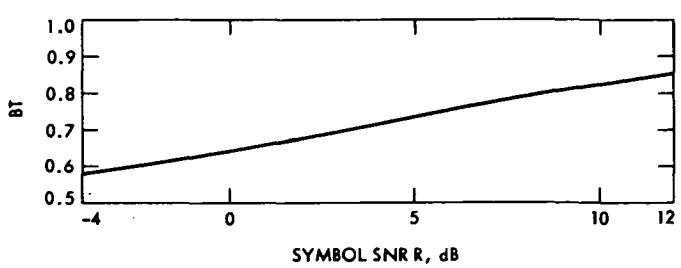

Fig. 3. Finding the optimal presquaring filter bandwidth $2B$ for the squared-suppressed-carrier (SSC) receiver (T = symbol time)

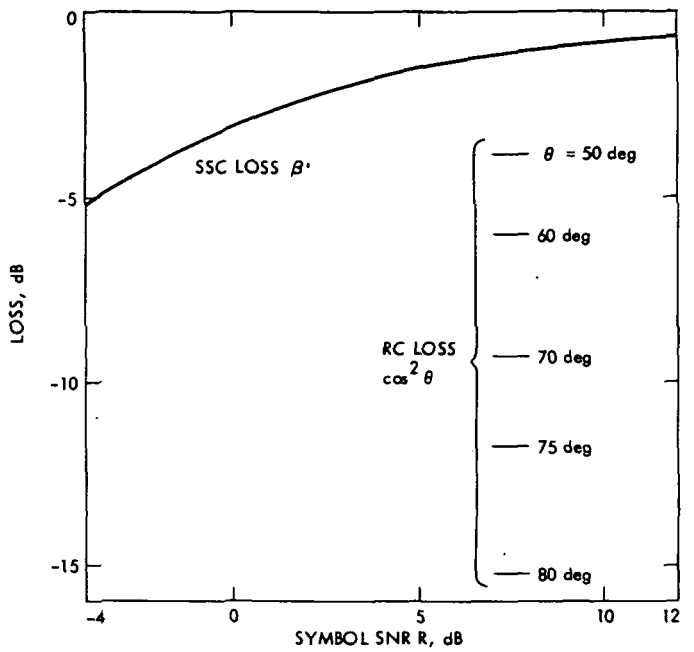

Fig. 4. Comparative losses of the SSC and residual-carrier (RC) receivers for several RC modulation indices θ

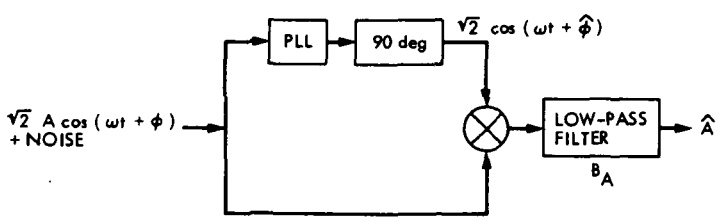

Fig. 5. Coherent amplitude detector

Appendix

S × S Components

Our aim here is to compute the discrete-spectrum and continuous-spectrum components of $s^2(t)$, where $s(t)$ is given by Eq. (9). To do this, we have found it expedient to approximate $d(t)$ by a waveform $d_N(t)$ of period NT, defined by

$$d_N(t) = d_j \quad (jT < t < (j+1)T), 0 \leqslant j \leqslant N-1)$$

$$d_N(t+NT) = d_N(t) \quad (\text{all } t) \tag{A-1}$$

where $d_0, \cdots, d_{N-1}$ are independent random variables taking the values 1 and -1 with probability 1/2. Then $d_N(t)$ has the discrete Fourier expansion

$$d_N(t) = \sum_{\lambda} D(\lambda) e^{i\lambda t} \tag{A-2}$$

where the λ spacing is $\Delta\lambda = 2\pi/(NT)$. Results will be obtained by letting $N \to \infty$.

Let $d(t)$ be replaced by $d_N(t)$. Ignoring DC components, we have

$$s^2(t) = \tfrac{1}{2} e^{i2\omega_c t} y(t) + \text{c.c.}$$

where

$$y(t) = \left[\sum_{\lambda} G(\omega_c + \lambda, t) D(\lambda) e^{i\lambda t}\right]^2$$

$$= \sum_{\omega} Y(\omega, t) e^{i\omega t} \tag{A-3}$$

The ω spacing is $\Delta\omega = 2\pi/(NT)$.

From Eq. (A-3),

$$Y(\omega, t) = \sum_{\omega} G(\omega_c + \lambda, t) G(\omega_c + \omega - \lambda, t)$$

$$\times D(\lambda) D(\omega - \lambda) \tag{A-4}$$

To compute the mean and variance of $Y(\omega, t)$ we need the second and fourth moments of the $D(\lambda)$ process. Let $\omega_1 = 2\pi/T$. From Eqs. (A-1) and (A-2),

$$D(\lambda) = \frac{e^{-i\lambda T} - 1}{-i\lambda T} \frac{1}{N} \sum_{j=0}^{N-1} d_j e^{-ij\lambda T}$$

$$(\lambda = k\omega_1/N, -\infty < k < \infty)$$

Then

$$E[D(\lambda_1)D(\lambda_2)] = \frac{(-1)^k}{N} \operatorname{sinc}(\tfrac{1}{2}\lambda_1 T) \operatorname{sinc}(\tfrac{1}{2}\lambda_2 T),$$

$$(\lambda_1 + \lambda_2 = k\omega_1)$$

$$= 0, \quad (\lambda_1 + \lambda_2 \neq 0 \bmod \omega_1) \tag{A-5}$$

$$\begin{aligned} E[D(\lambda_1)D(\lambda_2)D(\lambda_3)D(\lambda_4)] \\ = E[D(\lambda_1)D(\lambda_2)]E[D(\lambda_3)D(\lambda_4)] \quad &(\text{part 1}) \\ + E[D(\lambda_1)D(\lambda_3)]E[D(\lambda_2)D(\lambda_4)] \quad &(\text{part 2}) \\ + E[D(\lambda_1)D(\lambda_4)]E[D(\lambda_2)D(\lambda_3)] \quad &(\text{part 3}) \\ + \text{part 4} \end{aligned} \tag{A-6}$$

$$\text{part 4} = -\frac{2}{N^3} (-1)^k \operatorname{sinc}(\tfrac{1}{2}\lambda_1 T) \cdots \operatorname{sinc}(\tfrac{1}{2}\lambda_4 T),$$

$$(\lambda_1 + \lambda_2 + \lambda_3 + \lambda_4 = k\omega_1)$$

$$= 0, \quad (\lambda_1 + \lambda_2 + \lambda_3 + \lambda_4 \neq 0 \bmod \omega_1)$$

If $d(t)$ were a Gaussian process, then part 4 would be zero.

From Eq. (A-4),

$$EY(\omega, t) = \sum_{\lambda} G(\omega_c + \lambda, t) G(\omega_c + \omega - \lambda, t)$$

$$\cdot E[D(\lambda)D(\omega - \lambda)]$$

$$E|Y(\omega,t)|^2 = \sum_{\lambda} \sum_{\lambda'} G(\omega_c+\lambda,t)G(\omega_c+\omega-\lambda,t)$$

$$\cdot\, \bar{G}(\omega_c+\lambda',t)\bar{G}(\omega_c+\omega-\lambda',t)$$

$$\cdot\, E[D(\lambda)D(\omega-\lambda)D(-\lambda')D(\lambda'-\omega)] \qquad \text{A-7)}$$

where the bars mean complex conjugate. It is convenient to use the notation

$$F(\omega,\lambda,t) = G(\omega_c+\lambda,t)G(\omega_c+\omega-\lambda,t)$$

$$\times \operatorname{sinc}(\tfrac{1}{2}\lambda T)\operatorname{sinc}(\tfrac{1}{2}(\omega-\lambda)T) \qquad \text{(A-8)}$$

in terms of which the expectation of $Y(\omega, t)$ is:

$$EY(n\omega_1,t) = \frac{(-1)^n}{N}\sum_{\lambda} F(n\omega_1,\lambda,t) \quad (n=0,\pm1,\cdots)$$

$$EY(\omega,t) = 0 \quad (\omega \neq 0 \bmod \omega_1)$$

When Eq. (A-6) is used to compute the fourth moments in Eq. (A-7), the variance of $Y(\omega, t)$ takes the form

$$E\,|Y(\omega,t)|^2 = \sum_{i=1}^{4} \text{part } 1$$

where

$$\text{part } 1 = |EY(\omega,t)|^2$$

$$\text{part } 2 = \text{part } 3\,\frac{1}{N^2}\sum_{\lambda}\sum_{k=-\infty}^{\infty} F(\omega,\lambda,t)\bar{F}(\omega,\lambda+k\omega_1,t)$$

$$\text{part } 4 = -\frac{2}{N^3}\left|\sum_{\lambda} F(\omega,\lambda,t)\right|^2$$

We are going to interpret these by letting $N \to \infty$. When we do this, all the sums become integrals in which $1/N$ becomes $Td\lambda/(2\pi)$. Parts 2, 3, and 4 are all $O(1/N)$. For $\omega = n\omega_1$ we have in the limit

$$EY(n\omega_1,t) = (-1)^n \int_{-\infty}^{\infty} F(n\omega_1,\lambda,t)\,\frac{Td\lambda}{2\pi}$$

$$E|Y(n\omega_1,t)|^2 = |EY(n\omega_1,t)|^2$$

which says that the variances of the real and imaginary parts of $Y(n\omega_1, t)$ are zero, and hence

$$Y(n\omega_1,t) = (-1)^n \int_{-\infty}^{\infty} F(n\omega_1,\lambda,t)\,\frac{Td\lambda}{2\pi} \qquad \text{(A-9)}$$

a nonrandom quantity. When $n = 0$ we get Eq. (13) for the complex amplitude of the recovered doubled carrier; when $n \neq 0$ we get the spurs, which are expected to be smaller than the main peak because the functions $\operatorname{sinc}(\tfrac{1}{2}(\lambda - n\omega_1)T)$ are orthonormal. These discrete components necessarily have nonrandom amplitudes, otherwise $s^2(t)$ would have strange nonergodic properties.

Let $\omega \neq 0 \bmod \omega_1$. Then $EY(\omega, t) = 0$ and $E|Y(\omega, t)|^2 = O(1/N)$. This means that as $N \to \infty$ the discrete spectrum near ω tends to a spectral density. We can let $1/N = T\,\Delta\omega/(2\pi)$. Then

$$S_{ss}(2\omega_c+\omega,t) = 1/4 \lim_{\Delta\omega\to 0} \frac{E|Y(\omega,t)|^2}{\Delta\omega/(2\pi)}$$

$$= \frac{T}{2}\int_{-\infty}^{\infty} F(\omega,\lambda,t)\sum_{k=\infty}^{\infty}\bar{F}(\omega,\lambda+k\omega_1,t)\,\frac{Td\lambda}{2\pi}$$

$$-\frac{T}{2}\left|\int_{-\infty}^{\infty} F(\omega,\lambda,t)\,\frac{Td\lambda}{2\pi}\right|^2 \qquad \text{(A-10)}$$

Define

$$\Phi(\omega,\lambda,t) = \sum_{k=-\infty}^{\infty} F(\omega,\lambda+k\omega_1,t)$$

which is an ω_1-periodic function of λ. In terms of it, Eq. (A-10) becomes

$$S_{ss}(2\omega_c+\omega,t) = \frac{T}{2}\int_0^{\omega_1} |\Phi(\omega,\lambda,t)|^2\,\frac{Td\lambda}{2\pi}$$

$$-\frac{T}{2}\left|\int_0^{\omega_1}\Phi(\omega,\lambda,t)\,\frac{Td\lambda}{2\pi}\right|^2$$

$$=\frac{T}{2}\int_0^{\omega_1}|\Phi(\omega,\lambda,t)-Z(\omega,t)|^2\,\frac{Td\lambda}{2\pi} \tag{A-11}$$

where

$$Z(\omega,t)=\int_{-\infty}^{\infty}F(\omega,\lambda,t)\,\frac{Td\lambda}{2\pi}=\int_0^{\omega_1}\Phi(\omega,\lambda,t)\,\frac{Td\lambda}{2\pi} \tag{A-12}$$

Note that $Z(n\omega_1, t) = (-1)^n Y(n\omega_1, t)$ $(n = 0, \pm 1, \cdots)$. The special case $\omega = 0$ gives Eq. (16) of the text for the S X S noise spectral density at $2f_c$.